

MATH 106 CALCULUS I FOR BIO. & SOC. SCI. FALL 2012

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Homework 1.

Please show all your work.

- (1) The functions f and g are defined below

$$f(x) = \sqrt{x^2 - 1},$$

$$g(x) = \frac{1}{x}.$$

Find explicit descriptions and the domains of the following functions.

(a) $f \circ g$,

(b) $g \circ f$,

(c) $f \circ f$,

(d) $g \circ g$.

- (2) Consider the functions

$$p(x) = \frac{x^3 - x}{x + 1},$$

$$q(x) = x^2 - x.$$

(a) Find the domains of $p(x)$ and $q(x)$.

(b) Show that $p(x) = q(x)$ for all $x \neq -1$.

(c) Are $p(x)$ and $q(x)$ the same functions? Please explain your answer.

(d) Provide a sketch for the graph of $p(x)$.

- (3) Let h be function defined by

$$h(x) = \frac{x - 1}{x + 1}.$$

(a) Show that h is a one to one function.

(b) Find the inverse function of h .

- (4) Solve the following equations

(a)

$$\ln(x^2 - 1) - \ln(x + 1) = 1$$

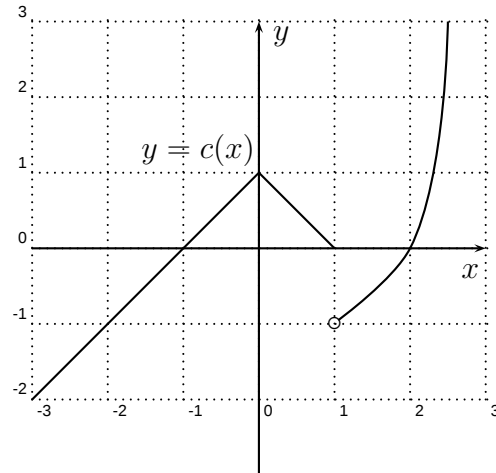
(b)

$$e^{\ln(x+1) - \ln(x-1)} = x$$

- (5) Let a be the function defined by $a(x) = \cos(\frac{\pi x}{4})$. Assume also that b is a one to one function such that

$$b(0) = -2, \quad b(1) = 0, \quad b(2) = 4, \quad b(3) = 5$$

and let c be the function whose graph is given below



Find the following quantities

- (a) $a \circ b^{-1}(0)$.
 - (b) $b \circ c(2)$.
 - (c) $c \circ a(4)$.
- (6) Radioactive isotopes are used in real life to determine the absolute age of fossils. The amount of a given radioactive isotope $W(t)$ present in a fossil after t years, is given by the equation

$$W(t) = W_0 e^{-\lambda t}.$$

In this case $\lambda = 0.0023104$ and W_0 is the initial quantity of the radioactive isotope. Find the half life of this isotope. Recall that the half life is defined to be the time that it takes for half of the radioactive material to decay.